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A Comprehensive Study on the Elzaki Transform of the Error Function

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ABSTRACT

Error function occurs frequently in probability, statistics, physics and many engineering problems like heat conduction problems, vibrating beams problems etc. In this article, we find the Elzaki transform of error function. In application section, some numerical applications of Elzaki transform of error function for evaluating the improper integral, which contain error function, are given.

Keywords: *Elzaki transform, Error function, Complementary error function, Improper integral.*
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I. INTRODUCTION

Integral transforms play a significant role for solving many advance problems of science and engineering such as radioactive decay problems, heat conduction problems, problem of motion of a particle under gravity, vibration problems of beam, electric circuit problems and population growth problems. Many researchers applied different integral transforms (Laplace transform [1-2], Fourier transform [2], Mahgoub transform [3-11, 41-43], Kamal transform [12-18, 44], Aboodh transform [19-24, 45-49], Mohand transform [25-28, 50-52], Elzaki transform [34-36, 53-55], Shehu transform [37-38, 56] and Sumudu transform [39, 57-58]) and solved differential equations, delay differential equations, partial differential equations, integral equations, integro-differential equations and partial integro-differential equations. Sudhanshu et al. [29-33, 40] discussed the comparative study of Mohand and other transforms (Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform and Mahgoub transform).

The solutions of many advanced engineering problems like Fick's second law, heat and mass transfer problems, vibrating beams problems contains error and complementary error function. When we solve these types of problems by using any integral transform then it is very necessary to knowing the integral transform of error function.
Mathematically error and complimentary error functions are defined by [59-64]

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \quad (1)$$

and

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt \quad (2)$$

In 2011, Elzaki [53] defined a new integral transform "Elzaki transform'' of the function $F(t)$ for $t \geq 0$ as

$$E\{F(t)\} = v\int_0^\infty F(t)e^{-t/v}dt = T(v), k_1 \leq v \leq k_2 \quad (3)$$

where operator E is called the Elzaki transform operator.

The goal of the present article is to determine Elzaki transform of error function and explain the advantage of Elzaki transform of error function for evaluating the improper integral, which contain error function.

II. SOME USEFUL PROPERTIES OF ELZAKI TRANSFORM

2.1 Linearity property [31]

If Elzaki transform of functions $F_1(t)$ and $F_2(t)$are$T_1(v)$and $T_2(v)$ respectively then Elzaki transform of

$[aF_1(t) + bF_2(t)]$ is given by $[aT_1(v) + bT_2(v)]$, where a, b are arbitrary constants.

Proof: By the definition of Elzaki transform, we have

$$E\{F(t)\} = v\int_0^\infty F(t)e^{-t/v}dt$$
$$\Rightarrow E\{aF_1(t) + bF_2(t)\} = v\int_0^\infty [aF_1(t) + bF_2(t)]e^{-t/v}dt$$
$$\Rightarrow E\{aF_1(t) + bF_2(t)\} = av\int_0^\infty F_1(t)e^{-t/v}dt + bv\int_0^\infty F_2(t)e^{-t/v}dt$$
$$\Rightarrow E\{aF_1(t) + bF_2(t)\} = aE\{F_1(t)\} + bE\{F_2(t)\}$$
$$\Rightarrow E\{aF_1(t) + bF_2(t)\} = aT_1(v) + bT_2(v),$$

where a, b are arbitrary constants.

2.2 Change of scale property

If Elzaki transform of function $F(t)$ is $T(v)$ then Elzaki transform of function $F(at)$is given by $\frac{1}{a^2}T(av)$.

Proof: By the definition of Elzaki transform, we have

$$E\{F(at)\} = v\int_0^\infty F(at)e^{-t/v}dt \quad (4)$$

Put $at = p \Rightarrow adt = dp$ in equation (4), we have

$$E\{F(at)\} = \frac{v}{a}\int_0^\infty F(p)e^{\frac{-p}{av}}dp$$
$$\Rightarrow E\{F(at)\} = \frac{1}{a^2}.av\int_0^\infty F(p)e^{\frac{-p}{av}}dp$$
$$\Rightarrow E\{F(at)\} = \frac{1}{a^2}T(av)$$

2.3 Shifting property

If Elzaki transform of function $F(t)$ is $T(v)$ then Elzaki transform of function $e^{at}F(t)$is given by $(1-av)T\left(\frac{v}{(1-av)}\right)$.

Proof: By the definition of Elzaki transform, we have

$$E\{e^{at}F(t)\} = v\int_0^\infty e^{at}F(t)e^{-t/v}dt = v\int_0^\infty F(t)e^{-\left(\frac{1}{v}-a\right)t}dt$$
$$= v\int_0^\infty F(t)e^{-\left[\frac{t}{\left(v/(1-av)\right)}\right]}dt$$
$$= \frac{v}{(1-av)}(1-av)\int_0^\infty F(t)e^{-\left[\frac{t}{\left(v/(1-av)\right)}\right]}dt = (1-av)T\left(\frac{v}{(1-av)}\right)$$

2.4 Elzaki transform of the derivatives of the function $F(t)$ [31, 35-36]

If $E\{F(t)\} = T(v)$ then

a) $E\{F'(t)\} = \frac{1}{v}T(v) - vF(0)$
b) $E\{F''(t)\} = \frac{1}{v^2}T(v) - F(0) - vF'(0)$

2.5 Elzaki transform of integral of a function $F(t)$

If $E\{F(t)\} = T(v)$ then $E\left\{\int_0^t F(t)dt\right\} = vT(v)$

Proof: Let $G(t) = \int_0^t F(t)dt$. Then $G'(t) = F(t)$and $G(0) = 0$.

Now by the property of Elzaki transform of the derivative of function, we have

$E\{G'(t)\} = \frac{1}{v}E\{G(t)\} - vG(0) = \frac{1}{v}E\{G(t)\}$
$\Rightarrow E\{G(t)\} = vE\{G'(t)\} = v\,E\{F(t)\}$
$\Rightarrow E\{G(t)\} = vT(v)$
$\Rightarrow E\left\{\int_0^t F(t)dt\right\} = vT(v)$

2.6 Elzaki transform of function tF(t)
If $E\{F(t)\} = T(v)$ then $E\{tF(t)\} = \left[v^2\frac{d}{dv} - v\right]T(v)$

Proof: By the definition of Elzaki transform, we have

$$E\{F(t)\} = v\int_0^\infty F(t)e^{-t/v}dt = T(v)$$
$$\Rightarrow \frac{d}{dv}T(v) = \int_0^\infty F(t)e^{-t/v}dt + v\int_0^\infty (-t)\frac{(-1)}{v^2}F(t)e^{-t/v}dt$$
$$\Rightarrow \frac{d}{dv}T(v) = \frac{1}{v}.v\int_0^\infty F(t)e^{-t/v}dt + \frac{1}{v^2}.v\int_0^\infty tF(t)e^{-t/v}dt$$
$$\Rightarrow \frac{d}{dv}T(v) = \frac{1}{v}T(v) + \frac{1}{v^2}E\{tF(t)\}$$
$$\Rightarrow E\{tF(t)\} = v^2\left[\frac{d}{dv} - \frac{1}{v}\right]T(v) = \left[v^2\frac{d}{dv} - v\right]T(v)$$

2.7 Convolution theorem for Elzaki transforms [31, 34]
If Elzaki transform of functions $F_1(t)$ and $F_2(t)$are$T_1(v)$and $T_2(v)$ respectively then Elzaki transform of their convolution $F_1(t) * F_2(t)$ is given by

$$E\{F_1(t) * F_2(t)\} = \frac{1}{v}E\{F_1(t)\}E\{F_2(t)\}$$
$\Rightarrow E\{F_1(t) * F_2(t)\} = \frac{1}{v}T_1(v)T_2(v)$, where $F_1(t) * F_2(t)$ is defined by
$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\,F_2(x)dx = \int_0^t F_1(x)\,F_2(t-x)dx$$

III. ELZAKI TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [31, 34-36]

Table: 1

S.N.	$F(t)$	$E\{F(t)\} = T(v)$
1.	1	v^2
2.	t	v^3
3.	t^2	$2!\,v^4$
4.	$t^n, n \in N$	$n!\,v^{n+2}$
5.	$t^n, n > -1$	$\Gamma(n+1)v^{n+2}$
6.	e^{at}	$\frac{v^2}{1-av}$
7.	$sinat$	$\frac{av^3}{1+a^2v^2}$
8.	$cosat$	$\frac{v^2}{1+a^2v^2}$
9.	$sinhat$	$\frac{av^3}{1-a^2v^2}$
10.	$coshat$	$\frac{v^2}{1-a^2v^2}$

IV. SOME IMPORTANT PROPERTIES OF ERROR AND COMPLEMENTARY ERROR FUNCTIONS

4.1 The sum of error and complementary error functions is unity

$\mathrm{erf}(x) + \mathrm{erfc}(f) = 1$

Proof: we have $\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2} dt = 1$$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt + \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt = 1$$

$\Rightarrow \mathrm{erf}(x) + \mathrm{erfc}(f) = 1$

4.2 Error function is an odd function:

$\mathrm{erf}(-x) = -\mathrm{erf}(x)$

4.3 The value of error function at $x = 0$ is 0:

$\mathrm{erf}(0) = 0.$

4.4 The value of complementary error function at $x = 0$ is 1:

$\mathrm{erfc}(0) = 1.$

4.5 The domain of error and complementary error functions is $(-\infty, \infty)$.

4.6 $\mathrm{erf}(x) \to 1$ as $x \to \infty$.

4.7 $\mathrm{erfc}(x) \to 0$ as $x \to \infty$.

4.8 The value of error functions $\mathrm{erf}(x)$ for different values of x [60]

Table: 2

S.N.	x	$erf(x)$
1.	0.00	0.00000
2.	0.02	0.02256
3.	0.04	0.04511
4.	0.06	0.06762
5.	0.08	0.09008
6.	0.10	0.11246
7.	0.12	0.13476
8.	0.14	0.15695
9.	0.16	0.17901
10.	0.18	0.20094
11.	0.20	0.22270

V. ELZAKI TRANSFORM OF ERROR FUNCTION

By equation (1), we have

$$erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-x^2}dx$$
$$= \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}}\left[1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots \ldots\right]dx$$
$$= \frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3.1!} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \cdots \ldots\right]_0^{\sqrt{t}}$$
$$= \frac{2}{\sqrt{\pi}}\left[t^{\frac{1}{2}} - \frac{t^{\frac{3}{2}}}{3.1!} + \frac{t^{\frac{5}{2}}}{5.2!} - \frac{t^{\frac{7}{2}}}{7.3!} + \ldots\right] \quad (5)$$

Applying Elzaki transform both sides on equation (5), we get

$$E\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}}E\left\{\left[t^{1/2} - \frac{t^{3/2}}{3.1!} + \frac{t^{5/2}}{5.2!} - \frac{t^{7/2}}{7.3!} + \ldots\right]\right\} \quad (6)$$

Applying the linearity property of Elzaki transform on equation (6), we get

$$E\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}}\left[\Gamma(3/2)v^{5/2} - \frac{\Gamma(5/2)}{3.1!}v^{7/2} + \frac{\Gamma(7/2)}{5.2!}v^{9/2} - \frac{\Gamma(9/2)}{7.3!}v^{11/2} + \cdots\right]$$
$$= \frac{2}{\sqrt{\pi}}\Gamma(3/2)v^{5/2}\left[1 - \frac{1}{2}v + \frac{1.3}{2.4}v^2 - \frac{1.3.5}{2.4.6}v^3 + \cdots \ldots \ldots\right]$$
$$= v^{5/2}(1+v)^{-1/2} = \frac{v^{5/2}}{\sqrt{(1+v)}} \quad (7)$$

VI. ELZAKI TRANSFORM OF COMPLEMENTARY ERROR FUNCTION

We have, $erf(x) + erfc(f) = 1$

$$\Rightarrow erfc(f) = 1 - erf(x) \quad (8)$$

Applying Elzaki transform both sides on equation(8), we have

$$E\{erfc(f)\} = E\{1 - erf(x)\} \quad (9)$$

Applying the linearity property of Elzaki transform on equation(9), we get

$$E\{erfc(f)\} = E\{1\} - E\{erf(x)\}$$
$$\Rightarrow E\{erfc(f)\} = v^2 - \frac{v^{5/2}}{\sqrt{(1+v)}}$$
$$\Rightarrow E\{erfc(f)\} = \left[\frac{v^2\sqrt{(1+v)} - v^{5/2}}{\sqrt{(1+v)}}\right] \quad (10)$$

VII. APPLICATIONS

In this section, some applications are given in order to explain the advantage of Elzaki transform of error function for evaluating the improper integral, which contain error function.

7.1 Evaluate the improper integral $\mathrm{I} = \int_0^\infty e^{-t}\,\mathrm{erf}(\sqrt{t})\,dt$.

We have $E\{\mathrm{erf}(\sqrt{t})\} = v\int_0^\infty \mathrm{erf}(\sqrt{t})\,e^{-t/v}dt = \frac{v^{5/2}}{\sqrt{(1+v)}}$ (11)

Taking $v \to 1$ in above equation, we have

$$\mathrm{I} = \int_0^\infty e^{-t}\,\mathrm{erf}(\sqrt{t})\,dt = \frac{1}{\sqrt{2}}$$

7.2 Evaluate the improper integral $\mathrm{I} = \int_0^\infty te^{-3t}\,\mathrm{erf}(\sqrt{t})\,dt$.

We have $E\{erf(\sqrt{t})\} = \frac{v^{5/2}}{\sqrt{(1+v)}}$

$$\Rightarrow E\{t\,erf(\sqrt{t})\} = \left[v^2\frac{d}{dv} - v\right]\frac{v^{5/2}}{\sqrt{(1+v)}}$$

$$= \frac{v^2(5v^{3/2}+4v^{5/2})}{2(1+v)^{\frac{3}{2}}} - \frac{v^{7/2}}{\sqrt{(1+v)}} \qquad (12)$$

By the definition of Elzaki transform, we have

$$E\{t\,erf(\sqrt{t})\} = v\int_0^\infty t\,erf(\sqrt{t})\,e^{-t/v}dt \qquad (13)$$

Now by equations (12) and (13), we get

$$v\int_0^\infty t\,erf(\sqrt{t})\,e^{-t/v}dt = \frac{v^2(5v^{3/2}+4v^{5/2})}{2(1+v)^{\frac{3}{2}}} - \frac{v^{7/2}}{\sqrt{(1+v)}}$$

Taking $v \to \frac{1}{3}$ in above equation, we have

$$\frac{1}{3}\int_0^\infty te^{-3t}\,erf(\sqrt{t})\,dt = \frac{11}{432}$$

$$I = \int_0^\infty te^{-3t}\,erf(\sqrt{t})\,dt = \frac{11}{144}$$

7.3 Evaluate the improper integral $I = \int_0^\infty e^{-\left(\frac{1}{v}-2\right)t}\,erf(\sqrt{t})\,dt$.

We have $E\{erf(\sqrt{t})\} = \frac{v^{5/2}}{\sqrt{(1+v)}}$

Now by shifting theorem of Elzaki transform, we have

$$E\{e^{2t}\,erf(\sqrt{t})\} = (1-2v)\frac{\left[{}^{v}/_{(1-2v)}\right]^{5/2}}{\sqrt{1+\frac{v}{(1-2v)}}}$$

$$\Rightarrow E\{e^{2t}\,erf(\sqrt{t})\} = \frac{v^{5/2}}{(1-2v)\sqrt{(1-v)}} \qquad (14)$$

By the definition of Elzaki transform, we have

$$E\{e^{2t}\,erf(\sqrt{t})\} = v\int_0^\infty e^{2t}\,erf(\sqrt{t})\,e^{-t/v}dt$$

$$\Rightarrow E\{e^{2t}\,erf(\sqrt{t})\} = v\int_0^\infty e^{-\left(\frac{1}{v}-2\right)t}\,erf(\sqrt{t})\,dt \qquad (15)$$

Now by equations (14) and (15), we get

$$v\int_0^\infty e^{-\left(\frac{1}{v}-2\right)t}\,erf(\sqrt{t})\,dt = \frac{v^{5/2}}{(1-2v)\sqrt{(1-v)}}$$

$$\Rightarrow I = \int_0^\infty e^{-\left(\frac{1}{v}-2\right)t}\,erf(\sqrt{t})\,dt = \frac{v^{3/2}}{(1-2v)\sqrt{(1-v)}}.$$

7.4 Evaluate the improper integral $I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{u})du\right\}dt$.

We have $E\{erf(\sqrt{t})\} = \frac{v^{5/2}}{\sqrt{(1+v)}}$

Now by the property of Elzaki transform of integral of a function, we have

$$E\left\{\int_0^t erf(\sqrt{u})du\right\} = v\left[\frac{v^{5/2}}{\sqrt{(1+v)}}\right]$$

$$\Rightarrow E\left\{\int_0^t erf(\sqrt{u})du\right\} = \frac{v^{7/2}}{\sqrt{(1+v)}} \qquad (16)$$

By the definition of Elzaki transform, we have

$$E\left\{\int_0^t erf(\sqrt{u})du\right\} = v\int_0^\infty e^{-t/v}\left\{\int_0^t erf(\sqrt{u})du\right\}dt \qquad (17)$$

Now by equations (16) and (17), we get

$$v\int_0^\infty e^{-t/v}\left\{\int_0^t erf(\sqrt{u})du\right\}dt = \frac{v^{7/2}}{\sqrt{(1+v)}}$$

Taking $v \to 1$ in above equation, we have

$$I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{u})du\right\}dt = \frac{1}{\sqrt{2}}.$$

7.5 Evaluate the improper integral $I = \int_0^\infty e^{-2t}\left[\frac{d}{dt}erf(2\sqrt{t})\right]dt$.

We have $E\{erf(\sqrt{t})\} = \frac{v^{5/2}}{\sqrt{(1+v)}}$

Now by change of scale property of Elzaki transform, we have

$$E\{erf(2\sqrt{t})\} = \frac{1}{16}\left[\frac{(4v)^{5/2}}{\sqrt{(1+4v)}}\right]$$

$$\Rightarrow E\{erf(2\sqrt{t})\} = \frac{2(v)^{5/2}}{\sqrt{(1+4v)}}$$

Now using the property of Elzaki transform of derivative of a function, we have

$$E\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = \frac{1}{v}.\left[\frac{2(v)^{5/2}}{\sqrt{(1+4v)}}\right] - v.0$$

$$\Rightarrow E\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = \frac{2(v)^{3/2}}{\sqrt{(1+4v)}} \qquad (18)$$

By the definition of Elzaki transform, we have

$$E\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = v\int_0^\infty e^{-t/v}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt \qquad (19)$$

Now by equations (18) and (19), we get

$$v\int_0^\infty e^{-t/v}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{2(v)^{3/2}}{\sqrt{(1+4v)}}$$

Taking $v \to \frac{1}{2}$ in above equation, we have

$$\frac{1}{2}\int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{1}{\sqrt{6}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{2}{\sqrt{(6)}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \sqrt{\frac{2}{3}}.$$

7.6 Evaluate the improper integral $I = \int_0^\infty e^{-5t}\left[erf\sqrt{t} * erf\sqrt{t}\right]dt$.

By convolution theorem of Elzaki transform, we have

$$E\{erf\sqrt{t} * erf\sqrt{t}\} = \frac{1}{v}E\{erf(\sqrt{t})\}E\{erf(\sqrt{t})\}$$

$$= \frac{1}{v}\left[\frac{v^{5/2}}{\sqrt{(1+v)}}\right]\left[\frac{v^{5/2}}{\sqrt{(1+v)}}\right] = \frac{v^4}{(1+v)} \qquad (20)$$

Now by the definition of Elzaki transform, we have

$$E\{erf\sqrt{t} * erf\sqrt{t}\} = v\int_0^\infty e^{-t/v}\{erf\sqrt{t} * erf\sqrt{t}\}dt \qquad (21)$$

Now by equations (20) and (21), we get

$$v\int_0^\infty e^{-t/v}\left\{\mathrm{erf}\sqrt{t} * \mathrm{erf}\sqrt{t}\right\}dt = \frac{v^4}{(1+v)} \quad (22)$$

Taking $v \to \frac{1}{5}$ in above equation, we have

$$\frac{1}{5}\int_0^\infty e^{-5t}\left\{\mathrm{erf}\sqrt{t} * \mathrm{erf}\sqrt{t}\right\}dt = \frac{1}{750}$$

$$\Rightarrow I = \int_0^\infty e^{-5t}\left\{\mathrm{erf}\sqrt{t} * \mathrm{erf}\sqrt{t}\right\}dt = \frac{1}{150}.$$

VIII. CONCLUSIONS

In this article, we have successfully discussed the Elzaki transform of error function. The given numerical applications in application section show the advantage of Elzaki transform of error function for evaluating the improper integral, which contain error function.

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