

## DOMINATION NUMBER EXTENSIONS: FROM BASE GRAPHS TO LINE GRAPHS

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### ABSTRACT

Let  $G$  be a simple connected graph of order  $n$  and  $L(G)$  be its line graph. A subset  $S$  of  $V$  is called a dominating set of  $G$  if every vertex of  $V - S$  is adjacent to some vertex in  $S$ . The domination number  $\gamma(G)$  of  $G$  is the minimum cardinality taken over all dominating sets of  $G$ . In this paper, we characterize regular graphs and unicyclic graphs of odd order for which  $\gamma(G) + \gamma(L(G)) = n - 2$ .

**Keywords:** Domination Number, Line Graph, Unicyclic Graphs, Regular Graphs.

### I. INTRODUCTION

Let  $G = (V, E)$  be a connected graph of order  $n$  and size  $m$ . The undefined terms and notations can be found in [5]. In 1956, Nordhaus and Gaddum [12] gave the lower and upper bound for the sum and product of chromatic number of a graph and its complement. In 1972, Jaeger and Payan [6] proved the same for domination number. The line graph  $L(G)$  of a graph whose vertex set is  $E(G)$  and two vertices of  $L(G)$  are adjacent if and only if the corresponding edges are adjacent in  $G$ . The concept of edge domination was introduced by Mitchell and Hedetniemi [10]. A subset  $S'$  of  $E$  is called an edge dominating set of  $G$  if every edge not in  $S'$  is adjacent to some edge in  $S'$ . The edge domination number  $\gamma'(G)$  of  $G$  is the minimum cardinality taken over all edge dominating sets of  $G$ . The domination number of a line graph  $L(G)$  of a graph  $G$  is the same as an edge domination number of a graph, that is  $\gamma'(G) = \gamma(L(G))$ . Recently [11], the authors characterized lower and upper bound for the sum  $\gamma(G) + \gamma(L(G))$ . In this paper, we characterize  $\gamma(G) + \gamma(L(G)) = n - 2$  for regular graphs and unicyclic graphs of odd order.

### II. PRELIMINARY RESULTS

The following results are required for our main theorems.

**Theorem 2.1. ([4,13])** For a graph  $G$  with even order  $n$  and no isolated vertices,  $\gamma(G) = n / 2$  if and only if the components of  $G$  are the cycle  $C_4$  or the corona  $H \circ K_1$  for any connected graph  $H$ .

In [3] E. J. Cockayne, et al characterized connected graphs for which  $\gamma(G) = \lfloor n/2 \rfloor$ . For this characterization, they defined six classes of graphs by using the following families of graphs. Let

$$\mathcal{G}_1 = \{C_4\} \cup \{G : G = H \circ K_1, \text{ where } H \text{ is connected}\}$$

and

$$\mathcal{G}_2 = \mathcal{A} \cup \mathcal{B} - \{C_4\}$$

For any graph  $H$ ,  $\mathcal{S}(H)$  denote the set of connected graphs, each of which can be formed from  $H \circ K_1$  by adding a new vertex  $x$  and edges joining  $x$  to one or more vertices of  $H$ . Then define

$$\mathcal{G}_3 = \bigcup_H \mathcal{S}(H)$$

Figure : 1

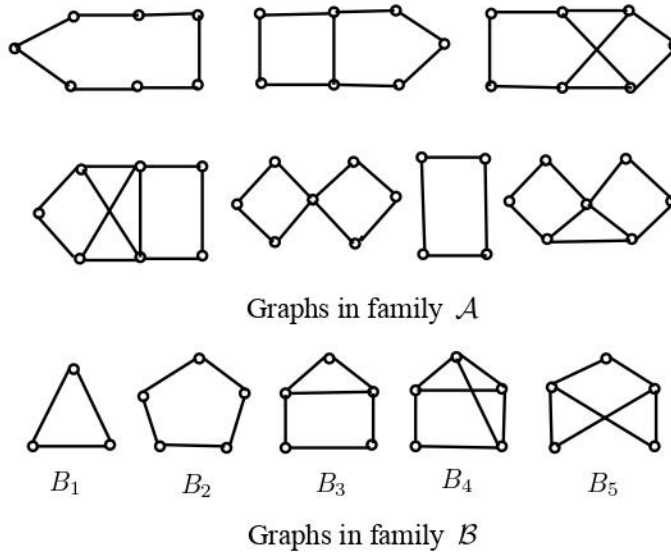


Fig. 1.

where the union is taken over all graphs  $H$ . Let  $y$  be a vertex of a copy of  $C_4$  and, for  $G \in \mathcal{G}_3$ , let  $\theta(G)$  be the graph obtained by joining  $G$  to  $C_4$  with the single edge  $xy$ , where  $x$  is the new vertex added in forming  $G$ . Then define

$$\mathcal{G}_4 = \{\theta(G) : G \in \mathcal{G}_3\}$$

Next, let  $u, v, w$  be a vertex sequence of a path  $P_3$ . For any graph  $H$ , let  $\mathcal{P}(H)$  be the set of connected graphs which may be formed from  $H \circ K_1$  by joining each of  $u$  and  $w$  to one or more vertices of  $H$ . Then define

$$\mathcal{G}_5 = \bigcup_H \mathcal{P}(H)$$

Let  $H$  be a graph and  $X \in \mathcal{B}$ . Let  $\mathcal{R}(H, X)$  be the set of connected graphs which may be formed from  $H \circ K_1$  by joining each vertex of  $U \subseteq V(X)$  to one or more vertices of  $H$  such that no set with fewer than  $\gamma(X)$  vertices of  $X$  dominates  $V(X) - U$ . Then define

$$\mathcal{G}_6 = \bigcup_{H, X} \mathcal{R}(H, X).$$

**Theorem 2.2.([3])** A connected graph  $G$  satisfies  $\gamma(G) = \lfloor n/2 \rfloor$  if and only if  $G \in \mathcal{G} = \bigcup_{i=1}^6 \mathcal{G}_i$ .

**Theorem 2.3.([14])** If  $G$  is a connected graph with  $\delta(G) \geq 3$ , then  $\gamma(G) \leq (3n)/8$ .

**Theorem 2.4.([2])** For any graph  $G$ ,  $\lfloor n/(1 + \Delta(G)) \rfloor \leq \gamma(G) \leq n - \Delta(G)$ .

**Theorem 2.5.([8])** If a graph  $G$  has no isolated vertices and  $\gamma(G) \geq 3$ , then  $\gamma(G) \leq (n + 1 - \delta)/2$ .

**Theorem 2.6.([1])** For any connected graph  $G$  of even order  $n$ ,  $\gamma'(G) = n/2$  if and only if  $G$  is isomorphic to  $K_n$  or  $K_{n/2, n/2}$ .

The graph obtained by identifying the centre of a subdivided star  $S(S_{1, k})$  with a vertex of  $C_3$  is denoted by  $C_{3, k}$ . The graph obtained by joining the centre of subdivided star  $S(S_{1, k})$  with a vertex of  $C_4$  by an edge  $e$  is denoted by  $C_{4, k}(e)$ .

**Theorem 2.7.([1])** Let  $G$  be a connected unicyclic graph. Then  $\gamma'(G) = \lfloor n/2 \rfloor$  if and only if  $G$  is isomorphic to  $C_4, C_5, C_7, C_{3, k}, C_{4, k}(e)$  for some  $k \geq 0$ .

**Lemma 2.8** Let  $H$  be any subgraph of  $G$ . Then  $\gamma(G) \leq \gamma(H) + \gamma(G - V(H))$ .

**Lemma 2.9** If  $H$  is a subgraph of  $G$ , then  $\gamma'(G) \leq \gamma'(H) + \gamma'(G - E(H))$ .

**Notation 2.10 ([7])** If  $G$  is a graph with vertex set  $V = \{u_1, u_2, \dots\}$ , then the graph obtained by identifying one of the end vertices of  $n_2$  copies of  $P_2$ ,  $n_3$  copies of  $P_3$  at  $u_1$ ,  $m_2$  copies of  $P_2$ ,  $m_3$  copies of  $P_3, \dots$  at  $u_2, \dots$  is denoted by  $G[u_1(n_2P_2, n_3P_3, \dots); u_2(m_2P_2, m_3P_3, \dots); \dots]$ .

### III. MAIN RESULTS

**Theorem 3.1** Let  $G$  be a connected unicyclic graph of odd order  $n \geq 5$ . Then  $\gamma(G) + \gamma(L(G)) = n - 2$  if and only if  $G$  is isomorphic to one of the graphs  $G_1, G_2, \dots, G_{29}$  given in Figure 2.

**Proof:** Let  $G$  be a connected unicyclic graph of odd order  $n \geq 5$ . If  $\gamma(G) + \gamma(L(G)) = n - 2$ , then we have the following two cases.

**Case: 1**  $\gamma(G) = (n - 3)/2$  and  $\gamma'(G) = (n - 1)/2$ .

By Theorem 2.7,  $G$  isomorphic to  $C_5, C_7, C_{3,k}, C_{4,k}(e)$  for some  $k \geq 0$ . But  $\gamma(G) = (n - 1)/2$  for these graphs.

**Case: 2**  $\gamma(G) = (n - 1)/2$  and  $\gamma'(G) = (n - 3)/2$ .

By Theorem 2.2,  $G \in \bigcup_{i=2}^6 \mathcal{G}_i$ . If  $G \in \mathcal{G}_2$ , then it is easy to verify that  $\gamma(L(G)) = \gamma'(G) = (n - 1)/2$  for these graphs.

**Subcase: 2.1**  $G \in \mathcal{G}_3$

If  $H$  is connected, then by Lemma 2.9,  $\text{diam}(H) = 1$  or  $2$  and so  $H$  is either  $K_2$  or star or  $C_3$  or  $C_4$  or  $C_5$  or  $C_3[u(kP_2)]$ . If  $H$  is a star, then  $x$  is adjacent to exactly two vertices of  $H$ . Hence  $G$  is isomorphic to  $G_1$  or  $G_2$  which satisfy the hypothesis. If  $H = C_3$  or  $C_4$  or  $C_5$  or  $C_3[u(kP_2)]$ , then  $x$  is adjacent to exactly one vertex of  $H$ . When  $H = C_4$  or  $C_5$  or  $C_3[u(kP_2)]$ , we observe that,  $\gamma'(G) = (n - 5)/2 \neq (n - 3)/2$ . If  $H = C_3$ , then  $G$  is isomorphic to  $G_3$  which satisfy the hypothesis. If  $H$  is disconnected, let  $H_1, H_2, \dots, H_s$  be the components of  $H$ . Clearly exactly one component, say  $H_i$  is nontrivial and  $\text{diam}(H_i) = 1$  or  $2$  and other  $H_j$ 's ( $j \neq i$ ) are trivial. Then by the previous argument for  $H_i$ ,  $G$  is isomorphic to  $G_4$  or  $G_5$  or  $G_6$  which satisfy the hypothesis.

**Subcase 2.2**  $G \in \mathcal{G}_4$

If  $H$  is connected, then we observe that  $\text{diam}(H) = 1$  or  $2$ . By the definition of  $\mathcal{G}_4$ ,  $H$  must be either  $K_2$  or star. Hence  $G$  is isomorphic to  $G_7$  or  $G_8$  which satisfy the hypothesis. If  $H$  is disconnected, then exactly one of its components is non-trivial whose diameter is  $1$  or  $2$  and others are trivial. Hence  $G$  is isomorphic to  $G_9$  or  $G_{10}$  which satisfy the hypothesis.

**Subcase 2.3**  $G \in \mathcal{G}_5$

If  $H$  is connected, then by Lemma 2.9,  $\text{diam}(H) = 1$  or  $2$  and so  $H$  is a star or  $C_3$  or  $C_4$  or  $C_5$  or  $C_3[u(kP_2)]$ . If  $H$  is a star, then both  $u$  and  $w$  are adjacent to exactly one vertex of  $H$  (or)  $u$  and  $w$  are adjacent to two distinct vertices of  $H$ . Hence  $G$  is isomorphic to  $G_{11}, G_{12}$  or  $G_{13}$  which satisfy the hypothesis. If  $H = C_3$  or  $C_4$  or  $C_5$  or  $C_3[u(kP_2)]$ , then either  $u$  or  $w$  is a pendant vertex (say  $w$ ). When  $H = C_4, C_5$  or  $C_3[u(kP_2)]$ , we observe that,  $\gamma'(G) = (n - 5)/2 \neq (n - 3)/2$ . If  $H = C_3$ , then  $G \in G_6$ .

If  $H$  is disconnected, let  $H_1, H_2, \dots, H_s$  be the components of  $H$ . Then we have the following two cases.

**Case: 2.3.1** Either  $u$  or  $w$  is a pendant vertex (say  $w$ ).

Then exactly one component, say  $H_i$  is non-trivial and  $\text{diam}(H_i) = 1$  or  $2$  and other  $H_j$ 's ( $j \neq i$ ) are trivial. If  $H_i$  is a star, then  $u$  is adjacent to exactly two vertices of  $H_i$  and it is adjacent to each vertex of  $H_j = K_1$  ( $j \neq i$ ). Hence  $G$  is isomorphic to  $G_4$  or  $G_5$ . Clearly  $H_i = C_3$ ; otherwise  $\gamma'(G) < (n - 3)/2$ . If  $H_i = C_3$ , then  $u$  is adjacent to exactly one vertex to each component of  $H$ . Hence  $G$  is isomorphic to  $G_6$ .

**Case: 2.3.2** Both  $u$  and  $w$  are not pendant vertices.

Then  $\langle uvw \rangle$  is isomorphic to  $C_3$  or  $P_3$ . If  $\langle uvw \rangle$  is isomorphic to  $C_3$ , then  $u$  and  $w$  are adjacent to different components of  $H$  and  $H_i$  must be a tree. If  $H_i$  is trivial, then  $G$  is isomorphic to  $G_{14}$  for which  $\gamma'(G) = (n - 3)/2$ . If  $\text{diam}(H_i) \leq 2$ , then  $H_i$  is a star and  $G$  is isomorphic to  $G_{15}$  or  $G_{16}$  which satisfy the hypothesis. Now let  $\langle uvw \rangle$  be isomorphic to  $P_3$ . If  $H = C_3$  or  $C_4$  or  $C_5$  or  $C_3[u(kP_2)]$ , then by Lemma 2.9,  $\gamma'(G) < (n - 3)/2$ . Hence  $H_i$  must be a tree and note that  $\text{diam}(H_i) = 0$  or  $1$  or  $2$ . Since  $H_i$  is a tree, both  $u$  and  $w$  are adjacent to exactly one vertex of  $H_i$  (or)  $u$  and  $w$  are adjacent to two distinct vertices of  $H_i$ . If  $\text{diam}(H_i) = 0$ , ( $H_i = K_1$  for all  $i$ ), then  $G$  is isomorphic to  $G_{17}$ . If  $\text{diam}(H_i) \leq 2$ , then  $H_i$  is a star. Hence  $G$  must be one of the graphs  $G_{18}, G_{19}, G_{20}$  which satisfy  $\gamma'(G) = (n - 3)/2$ .

**Subcase 2.4**  $G \in \mathcal{G}_6$

By the definition of  $\mathcal{G}_6$ ,  $X$  must be  $C_3$  or  $C_5$ .

**Case 2.4.1:**  $X = C_3$ .

If  $H$  is connected, then  $H$  is a tree with  $\text{diam}(H) \leq 2$  and so  $H$  is either  $K_1$  or a star  $K_{1,r}$  ( $r \geq 1$ ). Clearly  $|V(U)|$  must be 1. If  $H = K_1$ , then  $G$  is isomorphic to  $C_{3,1}$  but  $\gamma'(G) = (n-1)/2 \neq (n-3)/2$ . If  $H$  is a star, then  $G$  is isomorphic to  $G_{21}$  or  $G_{22}$  which satisfy the hypothesis. If  $H$  is disconnected, then it is either totally disconnected or exactly one component, say  $H_i$  is of diameter at most 2 and other components  $H_j$ 's ( $j \neq i$ ) are trivial. It is clear that  $H = K_1$  or a star and  $|V(U)| = 1$  or 2. Suppose  $|V(U)| = 1$ . If  $H_i = K_1$ , then  $G$  is isomorphic to  $C_{3,k}$  but  $\gamma'(G) = (n-1)/2 \neq (n-3)/2$ . If  $H_i$  is a star, then  $G$  is isomorphic to  $G_{23}$  or  $G_{24}$ . Suppose  $|V(U)| = 2$ . If  $H_i = K_1$ , then  $G$  is isomorphic to  $G_{14}$ . If  $H_i$  is a star, then  $G$  is isomorphic to  $G_{15}$  or  $G_{16}$ .

Figure : 2

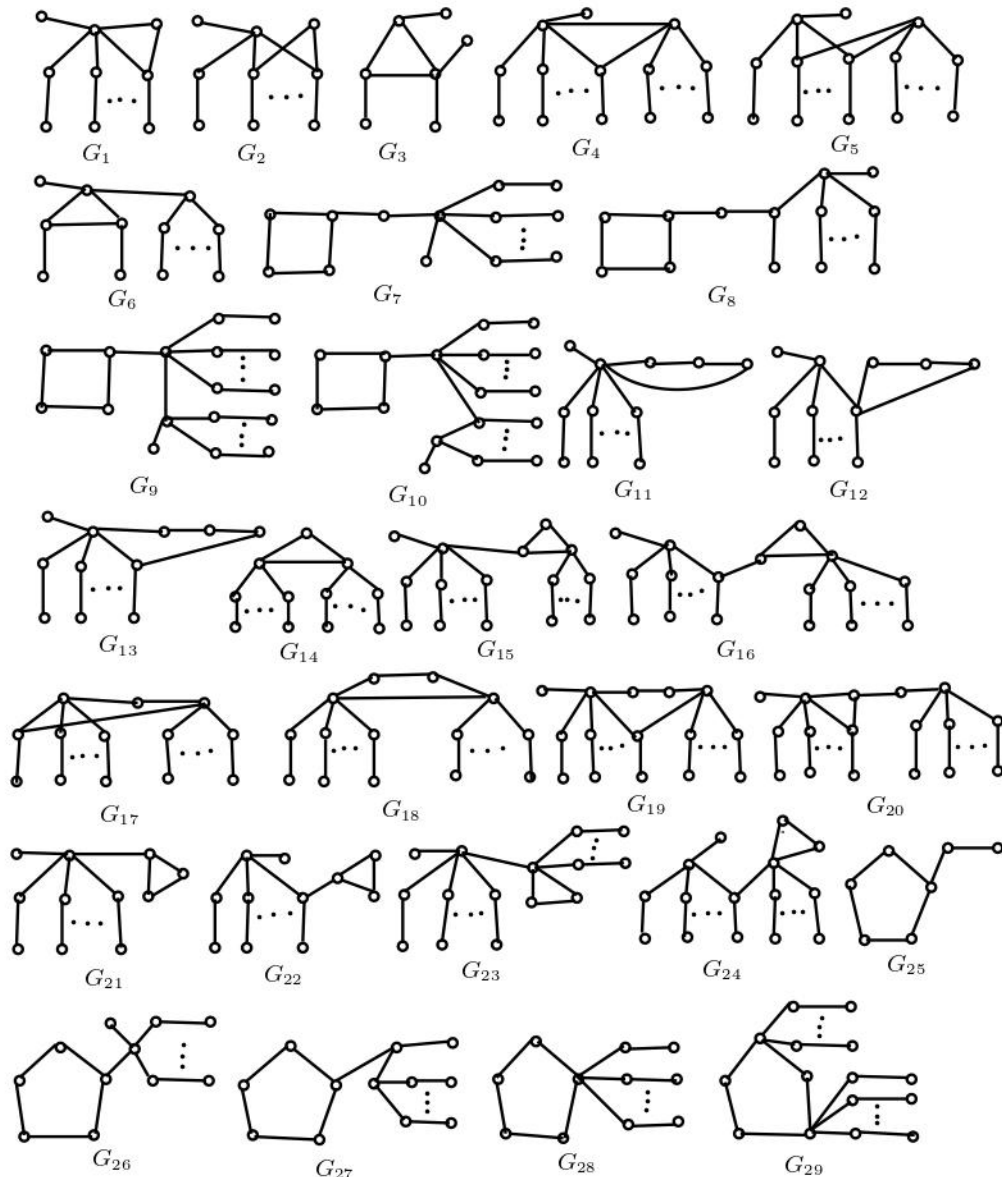


Fig. 2 Unicyclic Graphs satisfying  $\gamma(G) + \gamma(L(G)) = n - 2$

**Case 2. 4. 2:**  $X = C_5$ .

If  $H$  is connected, then  $H$  is a tree with  $\text{diam}(H) \leq 2$  and  $|V(U)| = 1$ . If  $H = K_1$ , then  $G$  is isomorphic to  $G_{25}$ . If  $H$  is a star, then  $G$  is isomorphic to  $G_{26}$  or  $G_{27}$ . If  $H$  is disconnected, then each component of  $H$  is trivial and  $|V(U)| = 1$  or 2. If  $|V(U)| = 1$ , then  $G$  is isomorphic to  $G_{28}$ .  $|V(U)| = 2$ , then  $G$  is isomorphic to  $G_{29}$ .

Conversely, if  $G$  is isomorphic to  $G_1, G_2, \dots, G_{29}$ , then it can be easily verified that  $\gamma(G) + \gamma(L(G)) = n - 2$ . The following lemma 3.2 is useful for Theorem 3.3.

**Lemma 3.2** If  $G$  is a connected 4-regular graph of order 11, then  $\gamma(G) = 3$ .

**Proof :** Let  $G$  be a connected 4-regular graph of order 11. Clearly  $\gamma(G) \geq 3$ . Let  $S$  be a  $\gamma$ -set of  $G$ . Let  $v$  be an arbitrary vertex of  $G$  and  $N(v) = \{v_1, v_2, v_3, v_4\}$ . Consider  $G' = G - N[v]$ . Since  $|V(G')| = 6$ , let  $V(G') = \{v_5, v_6, v_7, v_8, v_9, v_{10}\}$ . Clearly  $\gamma(G') \neq 1$ . Let  $S'$  be a  $\gamma$ -set of  $G'$ . If  $|S'| = 2$ , then  $S' \cup \{v\}$  is a  $\gamma$ -set of  $G$  and hence  $\gamma(G) = 3$ . Let

$E_1$  denote the set of edges between the vertices of  $G'$  and  $N(v)$  in  $G$ . Since  $G$  is 4-regular,  $|E_1| \leq 12$  and  $G'$  has at most two isolated vertices.

**Case 1:**  $G'$  has two isolated vertices.

Let  $v_5, v_6$  be the two isolated vertices in  $G'$  which are adjacent to all the vertices of  $N(v)$  in  $G$ . Then the remaining components of  $G'$  are either  $2K_2$  or  $K_3+{e}$  or  $C_4$  or  $C_4+{e}$  or  $K_{1,3}$  or  $P_4$  or  $K_4$ . Since  $|E_1| = 12$ ,  $G' = K_4 \cup 2K_1$  and  $v_1$  is adjacent to both  $v_5$  and  $v_6$ . Then  $S = \{v, v_1, v_7\}$  is a minimum dominating set of  $G$  and hence  $\gamma(G) \leq 3$  and so  $\gamma(G) = 3$ .

**Case 2 :**  $G'$  has one isolated vertex (say,  $v_5$ ).

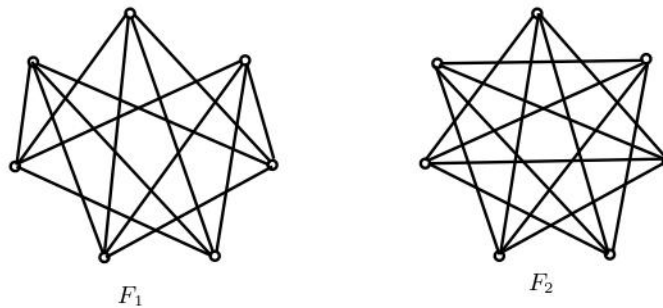
Then  $G'$  is isomorphic to  $H \cup K_1$ , where  $|V(H)| = 5$ . Since  $|E_1| \leq 12$ ,  $H$  is connected and has at most one pendant vertex and  $\Delta(H) = 3$  or 4. If  $\Delta(H) = 4$ , let  $d(v_6) = 4$  in  $G'$ . Then  $S = \{v, v_5, v_6\}$  is a minimum dominating set of  $G$  and hence  $\gamma(G) = 3$ . Now consider the case for  $\Delta(H) = 3$ . Let  $d(v_6) = 3$  in  $H$  and  $V(H - N[v_6]) = v_{10}$ . Clearly  $v_{10}$  must be adjacent to at least one of the vertices of  $N(v)$ , (say  $v_1$ ) in  $G$ . Then  $S = \{v, v_1, v_6\}$  is a minimum dominating set of  $G$  and hence  $\gamma(G) \leq 3$  and so  $\gamma(G) = 3$ .

**Case 3 :**  $G'$  has no isolated vertices.

Since  $|E_1| \leq 12$ ,  $G'$  is connected and is isomorphic to  $C_3 \cup K_1$ . Let  $V(C_3) = \{v_5, v_6, v_7\}$  and  $v_8, v_9, v_{10}$  be the corresponding pendant vertices of  $v_5, v_6, v_7$  in  $G'$ . Since at least one  $N(v)$ , say  $v_1$  must be adjacent to  $v_8, v_9, v_{10}$ . Then  $S = \{v, v_1, v_5\}$  is a minimum dominating set of  $G$  and hence  $\gamma(G) \leq 3$  and so  $\gamma(G) = 3$ . This completes the proof.

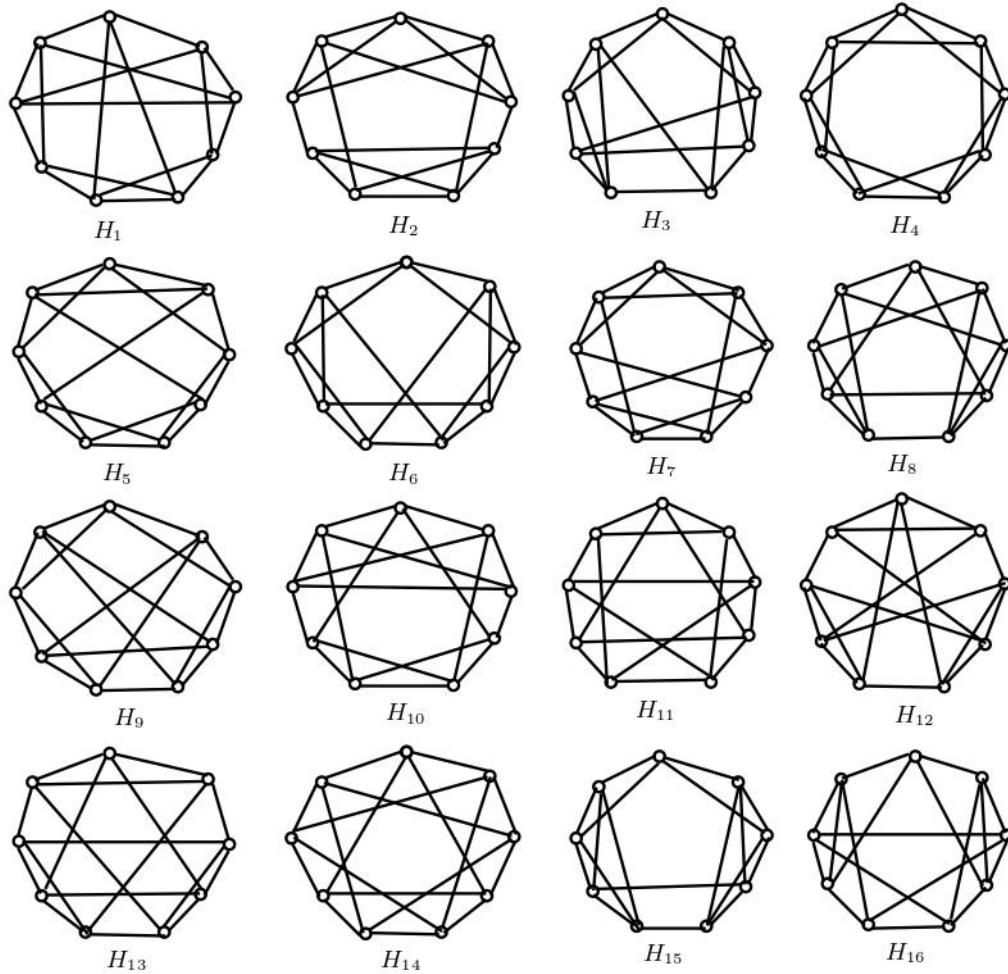
**Theorem 3.3** Let  $G$  be a connected  $k$ -regular graph of order  $n \geq 5$ . Then  $\gamma(G) + \gamma(L(G)) = n - 2$  if and only if  $G$  is isomorphic to either  $K_5, K_6, K_{4,4}$  or any one of the graphs  $F_1, F_2$  given in Figure 3.

**Figure : 3**



*Fig. 3 Regular graphs satisfying  $\gamma(G) + \gamma(L(G)) = n - 2$*

**Figure : 4**



**Fig. 4 All 4-regular connected graphs of order 9**

**Proof:** Assume that  $\gamma(G) + \gamma(L(G)) = n - 2$ . Then we have the following two cases.

**Case 1:**  $n$  is even

If  $\gamma(G) = n/2$  and  $\gamma'(G) = (n/2) - 2$ , then by Theorem 2.1 and hypothesis, no graph exists. If  $\gamma(G) = (n/2) - 2$  and  $\gamma'(G) = n/2$ , then by Theorem 2.6,  $G$  is isomorphic to  $K_n$  or  $K_{n/2, n/2}$ . If  $G = K_n$ , then  $\gamma(G) = 1 = (n/2) - 2$  which gives  $n = 6$  and hence  $G = K_6$ . If  $G = K_{n/2, n/2}$ , then  $\gamma(G) = 2 = (n/2) - 2$  which gives  $n = 8$  and hence  $G = K_{4,4}$ .

**Case 2:**  $n$  is odd

If  $\gamma(G) = (n-1)/2$  and  $\gamma'(G) = (n-3)/2$ , then by Theorem 2.2,  $G$  is either  $C_5$  or  $C_7$  for which  $\gamma'(G) = (n-1)/2 \neq (n-3)/2$ . Now we consider the case  $\gamma(G) = (n-3)/2$  and  $\gamma'(G) = (n-1)/2$ . If  $G$  is 2-regular, then  $G = C_n$ . We observe that  $\gamma'(C_n) = \lfloor n/3 \rfloor = (n-1)/2$  which gives  $n = 5$  and hence  $G = C_5$  but  $\gamma(G) = 2 \neq (n-3)/2$ . If  $k \geq 3$ , then by Theorem 2.3,  $n \leq 12$ . Since  $n$  is odd,  $k$  must be even and by hypothesis,  $n \in \{5, 7, 9, 11\}$ . If  $n = 5$ , then  $G = K_5$  for which  $\gamma(G) = 1 = (n-3)/2$  and  $\gamma'(G) = 2 = (n-1)/2$ . If  $n = 7$ , then  $k$  must be either 4 or 6. If  $k = 6$ , then  $G = K_6$  for which  $\gamma(G) = 1 \neq (n-3)/2$ . If  $k = 4$ , then by [8], there are exactly two graphs  $F_1, F_2$  which satisfy the hypothesis. If  $n = 9$ , then  $k = 4$  or 6 or 8. If  $k = 4$ , then by [8], there are sixteen 4-regular graphs of order 9 (See Figure 4) and it is easy to see that no graph satisfies the hypothesis. If  $k = 6$ , then by Lemma 2.8,  $\gamma(G) < (n-3)/2$ , a contradiction. If  $k = 8$ , then  $G = K_9$  for which  $\gamma(G) = 1 \neq (n-3)/2$ . If  $n = 11$ , then  $k = 4$  or 6 or 8 or 10. If  $k = 6$  or 8 or 10, then it is easy to see that  $\gamma(G) < (n-3)/2$ . If  $k = 4$ , then by Lemma 3.2,  $\gamma(G) = 3 \neq (n-3)/2$ . Converse is obvious by verification.

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